

Discrete Mathematics Quiz 4

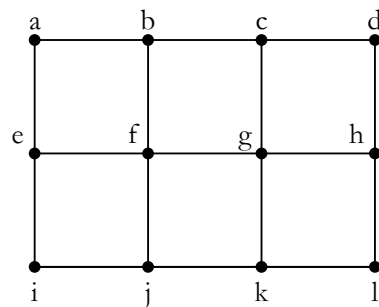
2023–2024 春夏学期

شركة ٥٠٥

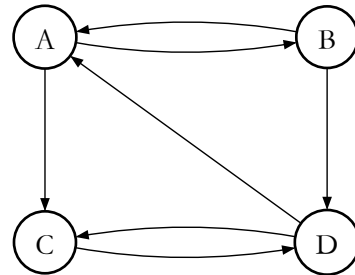
1. Fill in the blanks. (40%, 5% each)

- a) Write a proposition equivalent to $(p \wedge \neg q)$ that uses only p, q, \neg , and the connective \vee .
- b) Express the negations of the statement $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$ so that all negation symbols immediately precede predicates.
- c) If G is a planar connected graph with 20 vertices, each of degree 3, then G has _____ regions.
- d) Given a rectangular coordinate system in three-dimensional space, how many points are there whose three coordinate values are all rational numbers? _____

e) A dominating set of vertices in a simple graph is a set of vertices such that every other vertex is adjacent to at least one vertex of this set. A dominating set with the least number of vertices is called a minimum dominating set. Find the number of vertices of minimum dominating set for the given graph.



- f) Does the graph above have a Hamilton circuit? If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists. _____
- g) What is the worst-case time complexity of Dijkstra's algorithm for computing the shortest path between two points in a weighted graph with n vertices?
- h) How many distinct paths of length are there in the given graph? (Note: Cycles are allowed.) _____



- 2. There are three consecutive positive integers that are divisible by 5, 7, and 11 respectively (依次能被 5, 7, 11 整除). Find all the solutions. (6%)
- 3. (30%, 5% each) A standard deck of 52 cards consists of 4 suits, with each suit containing 13 cards corresponding to 13 values.
 - a) How many different poker hands of 5 cards that containing four cards of the same value? (xxxxy 牌型, 问有多少种可能取法)
 - b) How many different poker hands of 5 cards that containing 2 pairs (两个对子, 对子数字相同但花色可以不同) but not including 3 cards of the same value? (xxyyz 牌型, 问有多少种可能取法)

- c) How many playing cards do you need to take at random to ensure that there is a pair among them?
- d) How many playing cards do you need to take at random to ensure that there is a straight (顺子, 五张数字连续的牌) among them? (注: A-2-3-4-5 和 10-J-Q-K-A 都算顺子, 花色可以不同)
- e) Ignoring the differences in suits, how many different ways are there for taking out 3 cards? (把数字相同但花色不同的牌视为不可分辨)
- f) Put these 52 cards into a grid with 4 rows and 13 columns, and prove that by selecting one card from each column, you can always get all the 13 values. (提示: 可看作一个完全匹配问题)
4. Use generating functions to solve the recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$. (12%)
5. There are n red points and n green points on the plane, any three of them are not collinear. Please use induction to prove: These $2n$ points can be connected in pairs to form n non-intersecting line segments. Each line segment connects a red point and a green point. (12%)