

Discrete Mathematics Quiz 3

2023–2024 春夏学期

Xecades

1. $R = \{(a, a), (a, b), (b, d), (a, d)\}$ is a relation on $\{a, b, c, d\}$. Find the smallest relation containing the relation R that is:

- (a) (6%) partial order relation.
- (b) (6%) symmetric and transitive.

2. Given the undirected graph G as shown in Fig. 1.

(a) (6%) Use Kruskal's algorithm to find the minimum spanning tree of graph G . What is the order in which the edges are added to the minimum spanning tree?

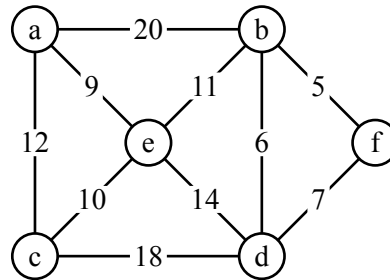


Fig. 1

(b) (6%) Using alphabetical ordering, find a spanning tree for this graph by depth-first search.

3. (6%) The frequencies of six characters are 0.09, 0.05, 0.2, 0.25, 0.3 and 0.11, respectively. If Huffman coding is used for optimal encoding, the average number of bits required to encode a character is _____.

4. (6%) How many leaves does a full 7-ary tree with 2024 vertices have?

5. (6%) Determine all positive integers r and s for which the complete bipartite graph $K_{r,s}$ is a tree.

6. (6%) Suppose $|A| = 4$. Find the number of different equivalence relations on A .

7. Answer these questions for the poset $(\{2, 3, 5, 6, 12, 20, 27, 36, 60\}, |)$.

- (a) (4%) Draw the Hasse diagram.
- (b) (2%) Find the maximal elements.
- (c) (2%) Is there a least element?
- (d) (2%) Find all upper bound of $\{2, 3\}$.

8. (10%) In the network below (Fig. 2), find a maximum flow from A to J , calculate its flow value, and prove that it is the maximum flow.

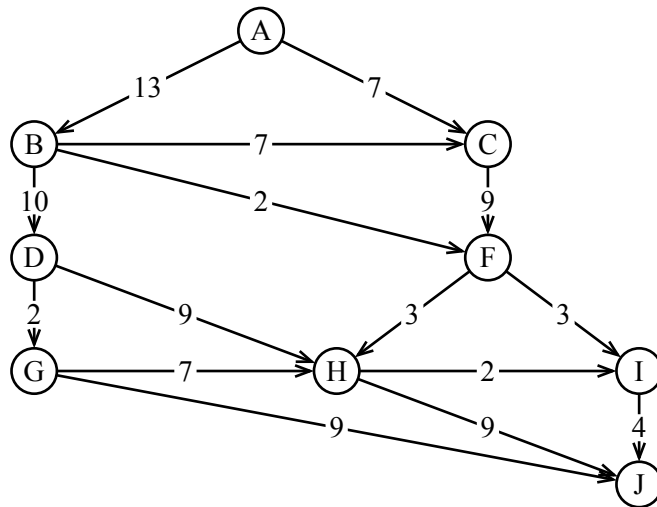


Fig. 2

9. (8%) Determine if the given pair of graphs (Fig. 3) is isomorphic. Give the reason.

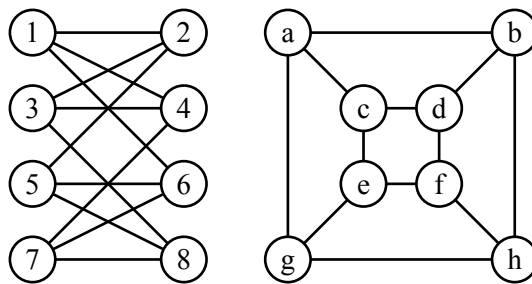


Fig. 3

10. Q_n is the graph with 2^n vertices representing bit strings of length n . An edge exists between two vertices that differ in exactly one bit position.
- (3%) Find the number of edges of Q_5 .
 - (3%) Find the chromatic number of Q_5 . Give the reason.
 - (6%) Determine if Q_5 has Hamilton circuit / path. Give the reason.
11. (12%) 8 students take a test with 8 true / false questions. It is known that no two students make exactly the same choice. Prove that we can remove one of the 8 questions, and still no two students make exactly the same choice.