Discrete Mathematics Quiz 3

2021 - 2022 春夏学期 郑文庭班

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1. Find the transitive closure of R on $\{a, b, c, d\}$,

where $R = \{(a, a), (b, a), (b, c), (c, a), (c, c), (c, d), (d, a), (d, c)\}$. (6%)

2.

(a) Find the smallest partial order relation R on $\{a, b, c, d, e, f\}$ that contains $(a, c), (c, c), (c, b), (c, d), (b, e), (b, f)$.

(b) Draw the Hasse diagram of R.

(c) List the maximal elements.

(d) List the minimal elements.

(e) Find the greatest element.

(f) Find the least element.

(g) Find the least upper bound of $\{d, e\}$.

(h) Use topological sorting to order the elements of the poset. (24%)

3. Find a minimum spanning tree for the weighted graph in Fig.1. You can just draw out the answer. (6%)
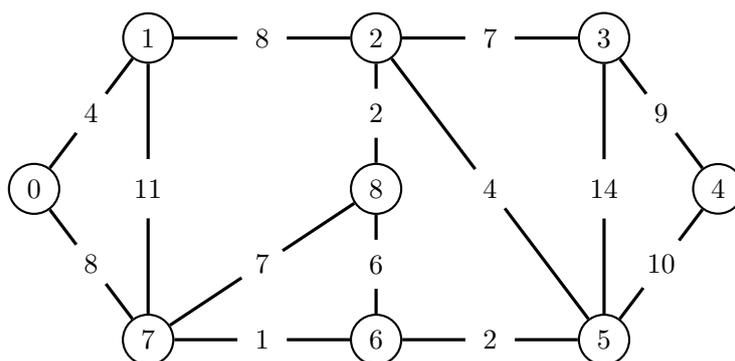

图 1: Fig.1

4. Use Dijkstra's Algorithm to find the shortest path length between the vertices 1 and 6 in the weighted graph in Fig.2. (10%)
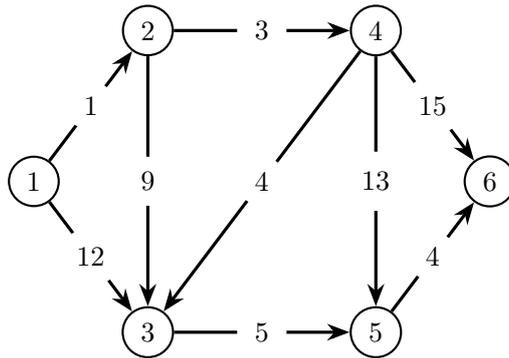

图 2: Fig.2

5. Use Huffman coding to encode these symbols with given frequencies: a: 0.15, b: 0.22, c: 0.26, d: 0.19, e: 0.08, f: 0.1. What is the average number of bits required to encode a character? (8%)

6. Determine all positive integers r and s for which $K_{r,s}$ is planar. Explain your answer. (8%)

7. In a round-robin tournament every player plays every other player exactly once and each match has a winner and a loser. There are total n players. Prove that we can sort the players in a certain order $p_1, p_2, \ldots, p_n$, so that p_1 beats p_2, p_2 beats p_3, …, and p_{n-1} beats p_n. (10%)

8. Fig.3 is the Petersen graph. (28%)
(a) Find the chromatic number of the Petersen graph.
(b) Determine whether the graph in Fig.4 is also a Petersen graph.
(c) Prove that the Petersen Graph is non-planar using Euler's formula.
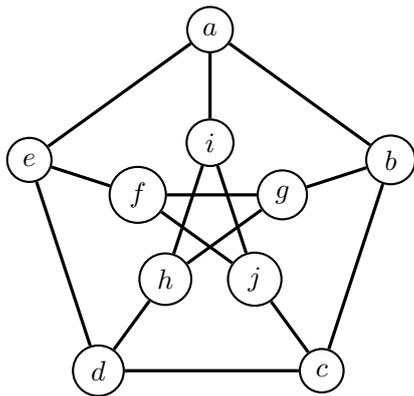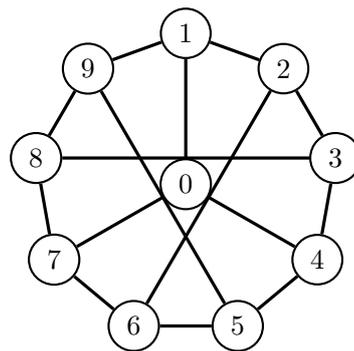(d) Determine whether the Petersen graph is Hamilton graph. Prove or disprove it.

图 3: Fig.3

图 4: Fig.4