Discrete Mathematics Quiz 1

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1. Determine whether the following statements are true or false. (30%)
    a) The following two propositions are logically equivalent:
$$p \to (q \to r), (p \to q) \to r$$
    b) If A, B, C are sets, then $A \oplus (B + C) = (A \oplus B) \oplus C$.
    c) $8 + 3 = 9$ iff $8 - 3 = 7$.
    d) The set of positive real numbers less than 1 with decimal representations consisting only of 6s and 8s is uncountable.
    e) The set of real numbers that are solutions of quadratic equations $ax^2 + bx + c = 0$, where a, b, c are integers, is countable.
    f) The time complexity of a linear search to find the smallest number in a list of n numbers is $\Theta(n \log n)$.

2. Suppose the variable x represents students, y represents courses, $T(x, y)$ means "x is taking y.". Translate the statement into symbols. (10%)
    a) There is a course that is being taken by all students.
    b) No student is taking all courses.

3. Suppose $g : A \to B$ and $f : B \to C$ where $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, $C = \{2, 7, 10\}$, and f and g are defined by $g = \{(1, b), (2, a), (3, a), (4, b)\}$ and $f = \{(a, 10), (b, 7), (c, 2)\}$. Find $f \circ g$. (5%)

4. Write a proposition equivalent to $(p \wedge \neg q)$ using only p, q, and the connective $|$. (7%)
   (" $|$ " represents NAND. The proposition $p \mid g$ is true when either p or q, or both, are false; and it is false when both p and q are true)

5. a) Express the proposition formula $p \oplus (q \oplus r)$ in full disjunctive normal form. (7%)
    b) Express the proposition formula $p \oplus (q \oplus r)$ in full conjunctive normal form. (7%)

6. Put the functions below in order so that each function is big-O of the next function on the list. (7%)

$$f_1(n) = (1.01)^n \qquad f_2(n) = 10n! \qquad f_3(n) = (\log n)^3$$
$$f_4(n) = 2^n \qquad f_5(n) = \log \log n \qquad f_6(n) = 999n^2(\log n)^3$$
$$f_7(n) = \frac{n^4 + 1}{n^3 + 3} \qquad f_8(n) = n^3 + n(\log n)^2 \qquad f_9(n) = 9^{999}$$

7. Set $A = \{\lceil x \rceil + \lceil 2x \rceil + \lceil 3x \rceil \mid x \in R\}$, set $B = x \mid x$ is a positive integer less than $2024\}$, find the value of $\{A \cap B\}$ (10%)

8. Prove that if x^3 is irrational, then x is irrational. (10%)

9. Use induction to prove that: if $x > 0, y > 0$, then $\frac{x^n + y^n}{2} \geq \left(\frac{x+y}{2}\right)^n$ for all positive integers n.